Penrith Selective High School
Mathematics Extension 1
Trial HSC 2019

General Instructions:

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- No correction tape or white out to be used
- A reference sheet is provided with this paper
- In Questions 11–14, show relevant mathematical reasoning and/ or calculations

	Prelim	Series/ Induction	Calculus	Inverse Fns	Rates of Change Growth & decay	Prob.	Motion	Total
M/C	/4	/1	/2		/1	/1	/1	/10
Q11	/3	/3	/5			/2	/2	/15
Q12				/3	/5		/7	/15
Q13		/4		/8		/3		/15
Q14	/6			/4	/3		/2	/15
Total	/13	/8	/7	/15	/9	/6	/12	/70

Student Number:____________________________________

Teacher's Name:____________________________________

Section I (10 marks)

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the Green multiple-choice answer sheet for Questions 1–10.

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1. The polynomial $3x^3 + 5x^2 - 7x - 10$ has zeros α, β and γ.

   What is the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$?

   A. $\frac{7}{10}$

   B. $-\frac{7}{10}$

   C. $\frac{1}{2}$

   D. $-\frac{1}{2}$

2. Solve $6|x+3| - 2|x+1| = 0$.

   A. -4 or -2.5

   B. 4 or 2.5

   C. -4 or 2.5

   D. 4 or -2.5

3. For what value of *c* will the line $5x - 12y + c = 0$ be a tangent to the circle $x^2 + y^2 = 4$?

   A. 78

   B. 13

   C. 52

   D. 26

4. Using Simpson's Rule with three functional values evaluate $\int_0^1 \sin^{-1} x\, dx$

A. $\frac{7\pi}{18}$

B. $\frac{\pi}{9}$

C. $\frac{7\pi}{6}$

D. $\frac{7\pi}{36}$

5. The derivative of $\ln(\sin 2x)$ is:

A. $2\cot 2x$

B. $2\tan 2x$

C. $\cot 2x$

D. $\tan 2x$

6. What is the exact value of tan105°?

A. $2 - \sqrt{3}$

B. $2 + \sqrt{3}$

C. $-2 - \sqrt{3}$

D. $-2 + \sqrt{3}$

7. Three men and three boys are seating at a round table.

What is the probability that they are alternating?

A. $\frac{1}{30}$

B. $\frac{3}{10}$

C. $\frac{1}{5}$

D. $\frac{1}{10}$

8. A particle moves so that at time *t* seconds, its displacement *x* metres from a fixed point *O* is given by $x = \pi + \sin(\frac{\pi}{3}t)$. When does the particle first return to its starting point?

A. 1.5

B. 3

C. 4.5

D. 6

9. Express 0.233 223 322 332……. as a fraction in its simplest form.

A. $\frac{233}{999}$

B. $\frac{233}{909}$

C. $\frac{212}{909}$

D. $\frac{232}{999}$

10. The rate of decay of a radioactive substance is proportional to the mass of the substance present at any time. If a quarter of the substance decomposed in 100 years, what percentage of the original amount remains after 500 years?

A. 9.8%

B. 8.8%

C. 31.6%

D. 23.7%

Section II – 60 marks

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section
- Answer each question in the appropriate writing booklet.
- Extra writing booklets are available.
- In Questions 11–14, your responses should include relevant mathematical reasoning and/ or calculations.

Question 11 (15 marks) Marks

(a) Solve $\frac{2x+1}{2x-1} > 2$ **3**

(b) How many terms are there in the geometric series:

$$3^{7m} + 3^{5m} + 3^{3m} + \ldots\ldots + 3^{(1-2k)m}?$$ **3**

(c) From the digits 1, 2, 3, 4 and 5 a five digit number is formed.

What is the probability that it is divisible by 2? **2**

(d) A particle moves with a constant acceleration of -3m/s^2.

If the velocity is -12m/s at two seconds, find the velocity when t = 3. **2**

(e) Find the primitive of the following functions:

(i) $1 + x^2$ **1**

(ii) $\frac{3x}{\sqrt{1+x^2}}$ **2**

(iii) $\frac{3x}{1+x^2}$ **2**

Question 12 (15 marks) Start a new booklet Marks

(a) Water is flowing into an inverted cone of base radius 6cm and perpendicular height 14cm, at a rate of $21\text{cm}^3/\text{s}$.

(i) Show that $\frac{dr}{dt} = \frac{9}{\pi r^2}$ **3**

(ii) Find the rate at which the surface area of the water is increasing when the depth of water is 8cm. **2**

(b) Find the exact value of $\cos(\sin^{-1}\frac{3}{4} + \cos^{-1}\frac{2}{3})$ giving justifications. **3**

(c) An object is thrown from the top of the library block 5m above the level ground. It is thrown at an angle of 30° to the horizontal with a velocity of 50m/s. (Take *g* as 10m/s^2)

(i) Write down the 6 equations of motion. **2**

(ii) Find the greatest height reached. **2**

(iii) A person standing on the ground 225m away from the foot of the library. Will the person be hit by the object? Give reasons for your answer. **3**

<u>Question 13</u> (15 marks) Start a new booklet Marks

(a) Prove by Mathematical Induction that $5^{2n} - 2^{3n}$ is always divisible by 17 for $n \geq 1$. **4**

(b) The area under the curve $y = \sin^{-1} x$ from $x = 0$ to $x = \frac{1}{2}$ bounded by the x-axis is rotated about the *y* –axis. Find the exact volume of the solid of revolution. **3**

(c) Evaluate exactly $\int_0^{\frac{1}{4}} \frac{10dx}{\sqrt{1-16x^2}}$ **3**

(d) An event has a probability of success of $\frac{1}{3}$ in a single trial. If *n* trials are conducted, find the least value of *n* for which the probability of obtaining exactly four successes exceeds triple the probability of obtaining exactly three successes. **3**

(e) Show that the derivative of $y = \tan^{-1}\sqrt{x}$ is $\frac{1}{2\sqrt{x}(1+x)}$ **2**

<u>Question 14</u> (15 marks) Start a new booklet Marks

(a) Neatly graph $y = \sin^{-1}(\cos x)$ for $0 \leq x \leq 3\pi$ **4**

(b) The tangent at *P* ($2ap$, ap^2) to $x^2 = 4ay$ meets the *x*-axis at *R*.
The tangent at *Q* ($2aq$, aq^2) meets the *y*-axis at *S*. Find the locus of the midpoint *RS*, given that *pq* = -2. (You may assume that the tangent is in the form $y = tx - at^2$). **3**

(c) A particle is moving on a straight line such that $x = 12 - 2^{1-t}$ where *x* is in metres and *t* is the time in seconds. Describe the motion. **2**

(d) *AOB* is a quadrant of a circle of radius 30cm.

P is a particle that moves on the arc *AB* about *O* at a constant rate.

It moves from *A* to *B* in 15 minutes.

$\angle AOP = \theta$

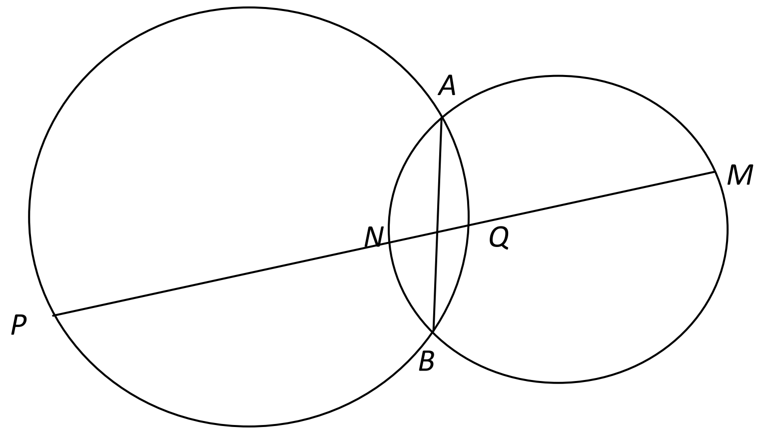
If *Y* is the total area of ΔOAP and ΔOBP, find the rate at which *Y* is changing when $\theta = \frac{\pi}{6}$. **3**

(e) *AB* is the common chord. **3**

PNXQM is a straight line.

X is the point of intersection of *AB* with *PNQM.*

Copy the diagram into your booklet. Prove that $\frac{PN}{NX} = \frac{MQ}{QX}$ Giving reasons.



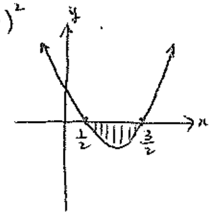
END OF EXAMINATION PAPER

Examination: Trial HSC 2019

Level: Maths Ext. 1

Year: 12

M/C 1. B 2. A 3. D 4. D 5. A 6. C 7. D 8. B 9. C 10. D

QUESTION: 11	Markers Comments
11 a) $\frac{2x+1}{2x-1} > 2$ $x \neq \frac{1}{2}$ Multiply both sides by $(2x-1)^2$ $(2x+1)(2x-1) > 2(2x-1)^2$ $4x^2-1 > 8x^2-8x+2$ $4x^2-8x+3 < 0$ (1) $(2x-3)(2x-1) < 0$ $\therefore \frac{1}{2} < x < \frac{3}{2}$ (2)	* $\frac{2x+1}{2x-1} > 2$ $2x+1 > 4x-2$ $3 > 2x$ $x < \frac{3}{2}$ 1 mark only
	
b) $a = 3^{7m}$, $r = 3^{-2m}$, $n = ?$, $T_n = 3^{(1-2k)m}$, G.P (1) $T_n = ar^{n-1}$ $3^{(1-2k)m} = 3^{7m} \cdot (3^{-2m})^{n-1}$ $3^{(1-2k)m} = 3^{7m-2m(n-1)}$ $(1-2k)m = 7m - 2m(n-1)$ (1) $2m(n-1) = 7m - m + 2km$ $n-1 = 3+k$ $\therefore n = 4+k$ terms (1)	* Alternative $a = 7m$, $d = -2m$, $T_n = (1-2k)m$, A.P. $T_n = a+(n-1)d$ $(1-2k)m = 7m + (n-1)(-2m)$ $1-2k = 7-2(n-1)$ $1-2k = 9-2n$ $2n = 2k+8$ $n = k+4$
c) Ending with 2 = 4! Ending with 4 = 4! $P(\text{divisible by } 2) = \frac{4!+4!}{5!}$ (1) $= \frac{2}{5}$ (1)	
d) $\ddot{x} = -3$ $\dot{x} = -3t + c$ $t=2$, $\dot{x} = -12$ } $c = -6$ (1) $\dot{x} = -3t - 6$ Sub $t = 3$, $\dot{x} = -9-6$ $= -15$ m/s (1)	

QUESTION: 11	Markers Comments
11 e) i) $\int (1+x^2)\,dx = x + \frac{x^3}{3} + c$ (1)	
ii) $\int \frac{3x}{\sqrt{1+x^2}}\,dx$ Let $u = 1+x^2$, $\frac{du}{dx} = 2x$, $\frac{1}{2}du = x\,dx$ $= \frac{3}{2}\int \frac{1}{\sqrt{u}}\,du$ (1) $= \frac{3}{2}(2u^{\frac{1}{2}}) + c$ $= 3\sqrt{1+x^2} + c$ (1)	* Some students got mixed up with $\sin^{-1}x$
iii) $\int \frac{3x}{1+x^2}\,dx$ $= \frac{3}{2}\int \frac{2x}{1+x^2}\,dx$ (1) $= \frac{3}{2}\log_e(1+x^2) + c$ (1)	* Some students got mixed with $\tan^{-1}x$.

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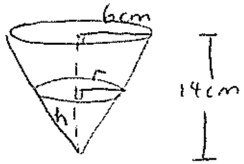
Q 12

a) i)



$\frac{r}{h} = \frac{6}{14}$

$6h = 14r$

$h = \frac{7r}{3}$ ✓ (1)

Given $\frac{dV}{dt} = 21$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$21 = \frac{7\pi}{3} r^2 \times \frac{dr}{dt}$

$\frac{dr}{dt} = 21 \times \frac{3}{7\pi r^2}$ ✓ (1)

$= \frac{9}{\pi r^2}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{7r}{3} \times \pi r^2$

$V = \frac{7\pi}{9} r^3$

$\frac{dV}{dr} = \frac{7\pi}{3} r^2$ ✓ (1)

* This question was very poorly attempted. Using similar triangles to find $h = \frac{7r}{3}$

ii) $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$= 2\pi r \times \frac{9}{\pi r^2}$

$= \frac{18}{r}$

when $h = 8$,

$8 = \frac{7r}{3}$

$r = 8 \times \frac{3}{7}$

$r = \frac{24}{7}$ ✓ (1)

$\frac{dA}{dt} = 18 \times \frac{7}{24}$

$= \frac{21}{4}$

$\therefore$ rising at $\frac{21}{4}$ cm²/s ✓ (1)

* poorly attempted

* Most students failed to find the correct 'r'.

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Q 12.

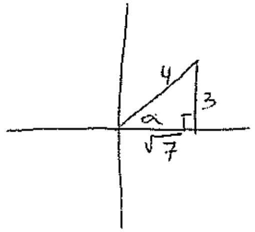
b) i) Let $\alpha = \sin^{-1}\frac{3}{4}$

$\sin\alpha = \frac{3}{4}$

$-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$

but $\sin\alpha > 0$

$\therefore 0 < \alpha < \frac{\pi}{2}$



* Need to justify why α and β is in first quadrant to get 1 mark

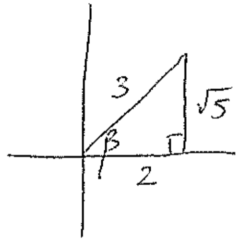
Let $\beta = \cos^{-1}\frac{2}{3}$

$\cos\beta = \frac{2}{3}$

$0 \le \beta \le \pi$

but $\cos\beta > 0$

$\therefore 0 < \beta < \frac{\pi}{2}$ ✓ (1)



$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{\sqrt{7}}{4} \times \frac{2}{3} - \frac{3}{4} \times \frac{\sqrt{5}}{3}$ ✓ (1)

$= \frac{\sqrt{7}}{6} - \frac{\sqrt{5}}{4}$

$= \frac{2\sqrt{7} - 3\sqrt{5}}{12}$ ✓ (1)

* Most students found the exact value but did not justify why α and β was in first quadrant, 1 mark was penalised.

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Q12

c) (i) $\ddot{x} = 0$

$\dot{x} = 50\cos 30° = 25\sqrt{3}$

$x = 25\sqrt{3}\,t$ ✓ (1)

$\ddot{y} = -10$

$\dot{y} = -10t + 25$

$y = -5t^2 + 25t + 5$ ✓ (1)

* Working out not necessary for this question.

ii) Max height when $\dot{y} = 0$

$-10t + 25 = 0$

$10t = 25$

$t = 2.5$ ✓ (1)

$y = -5(2.5)^2 + 25(2.5) + 5$

$y = 36.25\text{ m}$ ✓ (1)

* Well done.

* Carried forward marks was awarded if t was incorrect or equation 'y' was wrong in part (i).

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Q12.

c) iii) Let $y = 0$

$-5t^2 + 25t + 5 = 0$

$t^2 - 5t - 1 = 0$

$t = \dfrac{5 \pm \sqrt{25 - (4)(-1)}}{2}$

$= \dfrac{5 \pm \sqrt{29}}{2}$ ✓ (1)

but $t \geq 0$

$x = 25\sqrt{3}\,t$

$= 25\sqrt{3} \times \dfrac{5 + \sqrt{29}}{2}$

$= 224.85\text{ m}$ ✓ (1)

∴ The person will <u>not</u> be hit by the object since $224.85 < 225\text{ m}$ ✓ (1)

* common error: quadratic equation

$t = \dfrac{5 \pm \sqrt{24}}{2}$.

CFE marks was awarded if t was wrong.

* part (c) was the best attempted in Q12.

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Q13

a) Step 1: Prove true for $n=1$

$5^2 - 2^3 = 17$

$\therefore$ True for $n=1$ ✓ (1)

Step 2: Assume true for $n=k$

$5^{2k} - 2^{3k} = 17P$ where P is an integer

$\therefore 5^{2k} = 17P + 2^{3k}$

Must write ✓ (1)

Step 3: Prove true for $n=k+1$

$5^{2(k+1)} - 2^{3(k+1)} = 17Q$ where Q is integer

$LHS = 5^{2(k+1)} - 2^{3(k+1)}$

$= 5^2(5^{2k}) - 2^3(2^{3k})$

$= 25(17P + 2^{3k}) - 8(2^{3k})$ By assumption ✓ (1)

$= 425P + 17(2^{3k})$

$= 17(25P + 2^{3k})$ ✓ (1)

$= 17Q$

$= RHS$

Step 4: By the principle of mathematical induction it is true for all integer $n \geq 1$

* This question was very well done, marks are penalised if students didn't state 'P is an integer'!

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Q13

b) $y = \sin^{-1} x$ when $x=0$ $y=0$

$x = \sin y$ $x = \frac{1}{2}$ $y = \frac{\pi}{6}$

$$V = \pi \int_0^{\frac{\pi}{6}} \sin^2 y \, dy$$

$V = \pi r^2 h$
$= \pi \left(\frac{1}{2}\right)^2 \frac{\pi}{6}$
$= \frac{\pi^2}{24}$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{6}} (1 - \cos 2y) \, dy$$

$$= \frac{\pi}{2} \left[y - \frac{1}{2} \sin 2y \right]_0^{\frac{\pi}{6}} \quad (1)$$

$$= \frac{\pi}{2} \left[\left(\frac{\pi}{6} - \frac{1}{2} \sin \frac{\pi}{3}\right) - \left(0 - \frac{1}{2} \sin 0\right) \right]$$

$$= \frac{\pi}{2} \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right) \quad (1)$$

$$= \frac{\pi(2\pi - 3\sqrt{3})}{24} \text{ units}^3 \quad (1)$$

* Badly done $\therefore V = \frac{\pi^2}{24} - \frac{2\pi^2}{24} + \frac{3\sqrt{3}}{24} = \frac{3\sqrt{3}\pi - \pi^2}{24}$ c

* common errors, forgot to change $x = \frac{1}{2}$ to $y = \frac{\pi}{6}$.

* integrated $1 - \cos 2y$ incorrectly

* incorrect substitution.

* Most students only found volume of the area bounded by the y axis....

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Q13

c) $\int_0^{\frac{1}{4}} \frac{10}{\sqrt{1-16x^2}}\,dx = \frac{10}{4}\int_0^{\frac{1}{4}} \frac{1}{\sqrt{\frac{1}{16}-x^2}}$

$= \frac{10}{4}\left[\sin^{-1}\left(\frac{x}{\frac{1}{4}}\right)\right]_0^{\frac{1}{4}}$

$= \frac{10}{4}\left[\sin^{-1}(4x)\right]_0^{\frac{1}{4}}$ ✓ (1)

$= \frac{5}{2}(\sin^{-1}1 - \sin^{-1}0)$ ✓ (1)

$= \frac{5}{2}\left(\frac{\pi}{2} - 0\right)$

$= \frac{5\pi}{4}$ ✓ (1)

Well done!

d) $P(r=3) = {}^nC_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{n-3}$

$P(r=4) = {}^nC_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^{n-4}$

${}^nC_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^{n-4} > {}^nC_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{n-3} \times 3$ ✓ (1)

$\frac{n!}{4!(n-4)!} \times \frac{1}{3} > 3 \times \frac{n!}{3!(n-3)!} \times \frac{2}{3}$

$\frac{(n-3)!}{(n-4)!} > 24$ ✓ (1)

$n-3 > 24$

$n > 27$

Smallest value $n = 28$ ✓ (1)

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Q13

e) $y = \tan^{-1}\sqrt{x}$

$y = \tan^{-1} u$ Let $u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ ✓ (1)

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{1+u^2} \times \frac{1}{2\sqrt{x}}$

$= \frac{1}{1+(\sqrt{x})^2} \times \frac{1}{2\sqrt{x}}$ ✓ (1)

$= \frac{1}{2\sqrt{x}(1+x)}$

* This is a show question, hence you must show every step of working out, no matter how trivial it is.

* Marks was penalised if the answer $\frac{1}{2\sqrt{x}(1+x)}$ just magically appeared in the final step without any justification.

Examination:

Level:

Year:

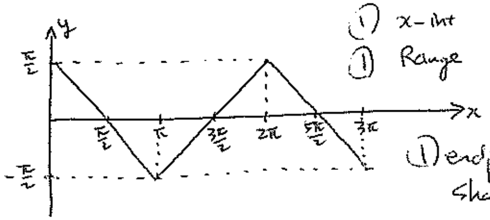
QUESTION: 14

14 a) $y = \sin^{-1}(\cos x)$

$= \sin^{-1}[\sin(\frac{\pi}{2} - x)]$

$= \frac{\pi}{2} - x$

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, x = all real numbers.



① x-int

① Range

① endpoints & shape

① Ruler.

b) Equation of tangent at P, $y = px - ap^2$

Sub $y=0$ for R,

$0 = px - ap^2$

$x = ap$ $\therefore R(ap, 0)$

Equation of tangent at Q, $y = qx - aq^2$

Sub $x=0$ for S,

$y = -aq^2$ $\therefore S(0, -aq^2)$ ①

$\therefore M\left(\frac{ap}{2}, -\frac{aq^2}{2}\right)$ ①

$x = \frac{ap}{2}$, $y = -\frac{aq^2}{2}$

$p = \frac{2x}{a}$ $q^2 = -\frac{2y}{a}$

$p^2 = \frac{4x^2}{a^2}$

$pq = -2$

$p^2q^2 = 4$

$\frac{4x^2}{a^2} \times \left(-\frac{2y}{a}\right) = 4$

$-8x^2y = 4a^3$

$y = -\frac{a^3}{2x^2}$ ①

Markers Comments

* Label the axes
* Scales on the x-axis need to be equally spaced
* Some students drew a curve instead of a straight line.

* Don't need to derive the equation of the tangent by differentiating the curve, it was given in the question. Just replace the parameter t by p and q
* Most students worked out the coordinates of the point R and P and the midpoint, but some struggled to get the cartesian equation of the locus by eliminating p and q

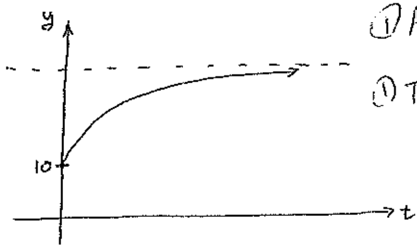
Examination:

Level:

Year:

QUESTION:

14 c) $x = 12 - 2^{1-t}$

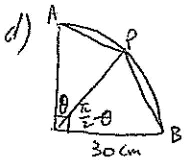


① Any two key points

① The other two key points

The particle was initially at $x=10$ to the right of the origin O, moving to the right. It slows down and approaches to $x=12$, but never actually reaches it or stops

d)



$\frac{d\theta}{dt} = \frac{\pi}{2}$ rad/15 min

$= \frac{\pi}{30}$ rad/min

$Y = \frac{1}{2}(30)^2\sin\theta + \frac{1}{2}(30)^2\sin(\frac{\pi}{2} - \theta)$

$= 450\sin\theta + 450\cos\theta$ ①

$\frac{dY}{d\theta} = 450\cos\theta - 450\sin\theta$

$\frac{dY}{dt} = \frac{dY}{d\theta} \times \frac{d\theta}{dt}$

$= 450(\cos\theta - \sin\theta) \times \frac{\pi}{30}$ ①

When $\theta = \frac{\pi}{6}$, $\frac{dY}{dt} = \frac{450\pi}{30}\left(\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\right)$

$= 15\pi\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$

$= \frac{15\pi}{2}(\sqrt{3} - 1)$ cm/min ①

Markers Comments

* Poorly done
* $\frac{d\theta}{dt} \ne \frac{360°}{15\text{ min}}$ or $6°$/min. The angle needs to be in radians
* Many students found the area of sectors using $\frac{1}{2}r^2\theta$ instead of the two triangles. Y = total area of the two Δ's
* 1 mark was given if you have at least attempted to apply the chain rule in terms of $\frac{dY}{d\theta}$ and $\frac{d\theta}{dt}$

Examination:

Level:

Year:

QUESTION: 14	Markers Comments
14 e) $(PX)(XQ) = (AX)(XB)$ (Products of intercepts of chords in a circle) Similarly $(MX)(XN) = (AX)(XB)$ (1) $\therefore (PX)(XQ) = (MX)(XN)$ $(PN + NX)(XQ) = (MQ + QX)(XN)$ $(PN)(XQ) + (NX)(XQ) = (MQ)(XN) + (QX)(XN)$ (1) $(PN)(XQ) = (MQ)(XN)$ Divide both sides by $(XN)(XQ)$ (1) $\therefore \frac{PN}{NX} = \frac{MQ}{QX}$	